

# 1. A Thick lens problem

$$M_{VV'} = \begin{pmatrix} 1 - P'D/n & -P - P' + \frac{2PP'D}{n} \\ \frac{D}{n} & 1 - \frac{PD}{n} \end{pmatrix}$$

where

$$P = -\frac{n-1}{R}$$

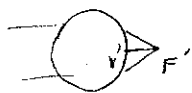
$$P' = +\frac{n-1}{R}$$

$$D = 2R$$

$$M_{VV'} = \begin{pmatrix} 1 - \frac{2(n-1)}{n} & -\frac{2(n-1)}{R} + \frac{2R(n-1)^2}{R^2 n} \\ \frac{2R}{n} & 1 - \frac{2(n-1)}{n} \end{pmatrix}$$

$$M_{VV'} = \begin{pmatrix} \frac{2-n}{n} & -\frac{2(n-1)}{R} \left(1 - \frac{n-1}{n}\right) \\ \frac{2R}{n} & \frac{2-n}{n} \end{pmatrix} = \begin{pmatrix} \frac{2-n}{n} & -\frac{2(n-1)}{nR} \\ \frac{2R}{n} & \frac{2-n}{n} \end{pmatrix}$$

(a)



$$\tilde{M} = \begin{pmatrix} 1 & 0 \\ S' & 1 \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ S & 1 \end{pmatrix} = \begin{pmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{pmatrix}$$

$\tilde{M}_{12} = M_{12}$   
 $\tilde{M}_{22} = -\frac{2(n-1)^2}{nR}$

$$\tilde{M}_{21} = M_{11}S' + M_{12}SS' + M_{21} + M_{22}S = 0 \quad \text{for infinite } S$$

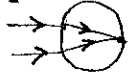
$$S \rightarrow \infty \quad S' = F'V = -\frac{M_{22}}{M_{12}} = -\frac{2-n}{n} \left( -\frac{nR}{2(n-1)} \right) = \frac{2-n}{n-1} \times \frac{R}{2}$$

$$F'V = \frac{2-n}{n-1} \times \frac{R}{2}$$

$$n = \frac{3}{2} = 1.5$$

$$(a) \quad F'V = \frac{0.5}{0.5} \times \frac{R}{2} = \frac{R}{2}$$

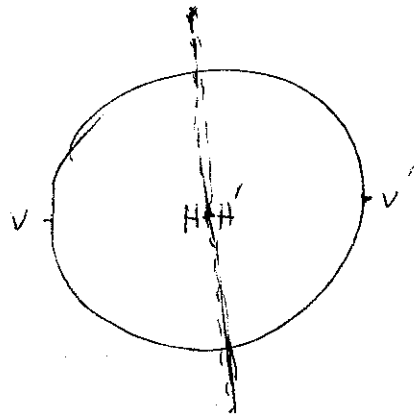
Note: For  $n=2$   
 $F'V=0$ , the rays are  
 focused into back  
 surface at  $V'$ .



$$(b) \quad P_e = P_{sys} = P_+ P'_- P P' \frac{D}{h} = \frac{2(n-1)}{R} \frac{(n-1)^2 \times 2R}{R^2 n}$$

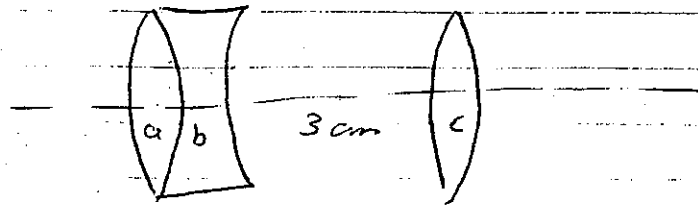
$$= \frac{2(n-1)}{nR}$$

$$D' = D = \frac{D'D}{-P_e n} = \frac{\frac{n-1}{R} \times 2R}{-\frac{2(n-1)}{nR} n} = -R$$



$$(c) \quad f' = \frac{1}{P} = \frac{nR}{2(n-1)}$$

2 (ICF # 3.19)



$$M_{\text{sys}} = M^c T_{bc} M^b M^a$$

For each lens (since  $R_1 = -R_2$ ) we have

$$M^j = \begin{pmatrix} 1 - \frac{n_i - 1}{n_j} \frac{D}{R} & \frac{2(1-n_i)}{R} - \frac{(1-n_i)^2}{R^2} \frac{D}{n_j} \\ \frac{D}{n_j} & 1 - \frac{n_j - 1}{n_i} \frac{D}{R} \end{pmatrix}$$

where  $R_i = +15 \text{ cm}$  for lenses a and c, and  $R = -15 \text{ cm}$  for lens b

$D = 0.5$  for all  $n_a = 1.5$   $n_b = 1.3$   $n_c = 1.6$

and  $T_{bc} = \begin{pmatrix} 1 & 0 \\ 3 \text{ cm} & 1 \end{pmatrix}$

\* Note: We have assumed here that lenses (a) and (b) are both ~~immersed~~ (surrounded) by air ( $n=1$ ). The result is the same as taking  $n=1$ ,  $n'=n_b$  for lens (a) - and  $n=1$ ,  $n'=1$  for lens (b). The former method is easier.

Using Math Cad:

$$M_{\text{sys}} = \begin{pmatrix} 0.699 & -0.097 \\ 4.015 & 0.87 \end{pmatrix}$$

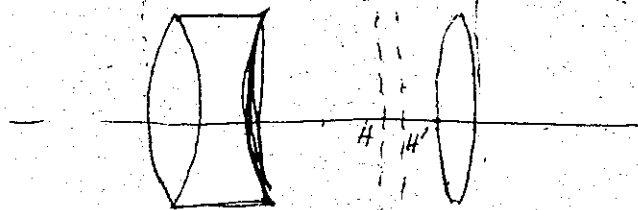
$$P_{\text{sys}} = -M_{12} = 0.097 \text{ ~~cm~~ cm}^{-1} = 9.77 \text{ m}^{-1}$$

D (diaptr)

\* Principal planes

$$D = \frac{1 - M_{11}}{M_{12}} = -3.1$$

$$D' = \frac{1 - M_{22}}{M_{12}} = -1.34$$



Note  $HH' = 4.5 - 4.44 = 0.06 \text{ cm}$

$$\underline{\underline{0.6 \text{ mm}}}$$

3 (KPF 3.56)

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad s + s' = L$$

$$\frac{1}{s} + \frac{1}{L-s} = \frac{1}{f} \Rightarrow s^2 - sL + Lf = 0$$

$$s = \frac{L \pm \sqrt{L^2 - 4Lf}}{2}$$

Note  $L^2 - 4Lf > 0$   $L > 4f$

$$\begin{aligned} \Rightarrow s_1 &= \frac{L + \sqrt{L^2 - 4Lf}}{2} \\ \Rightarrow s_2 &= \frac{L - \sqrt{L^2 - 4Lf}}{2} \end{aligned} \quad \left. \vphantom{\begin{aligned} \Rightarrow s_1 &= \frac{L + \sqrt{L^2 - 4Lf}}{2} \\ \Rightarrow s_2 &= \frac{L - \sqrt{L^2 - 4Lf}}{2} \end{aligned}} \right\} |s_1 - s_2| = d = \sqrt{L^2 - 4Lf}$$